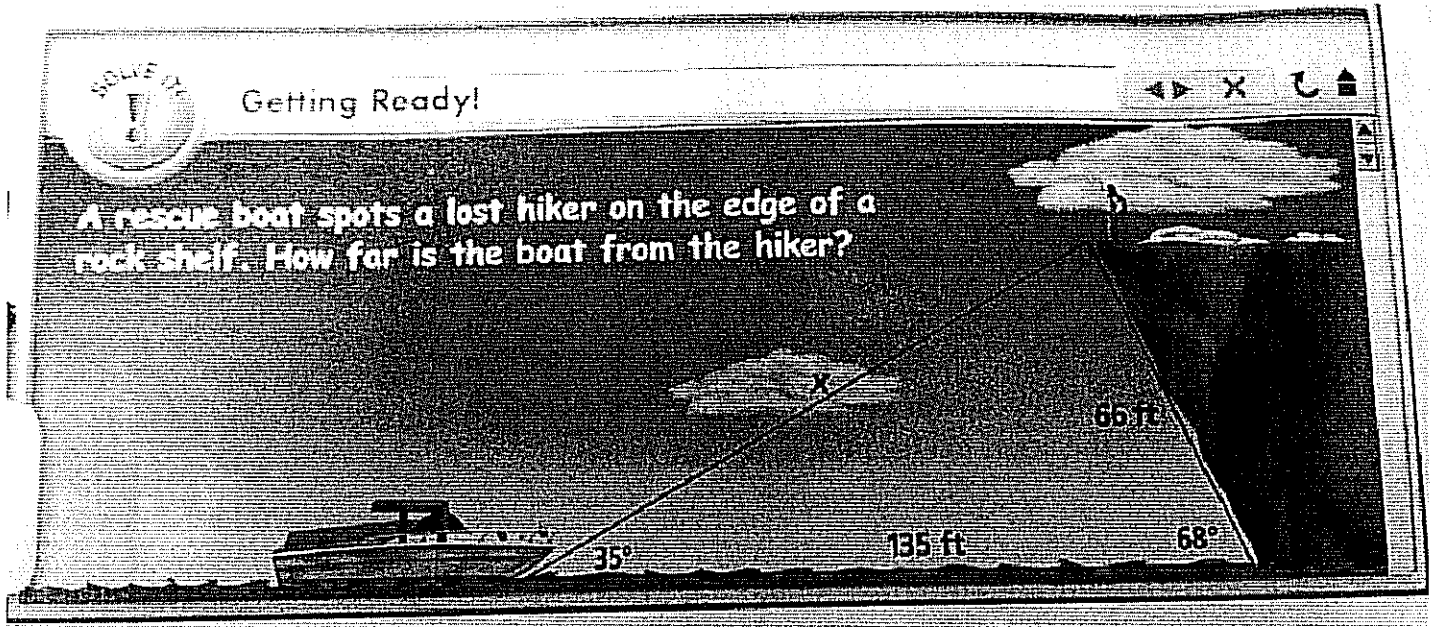


CC Triangle Supplements

Law of Sines and Law of Cosines

L8.5 (PCC) Law of Sines

The Law of Sines and the Law of Cosines are used to solve non-right triangles.



➤ The Law of Sines

For any $\triangle ABC$, let the lengths of the sides opposite angles A , B , and C be a , b , and c , respectively. Then the Law of Sines relates the sine of each angle to the length of the opposite side.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Proof:

$\triangle ACD$

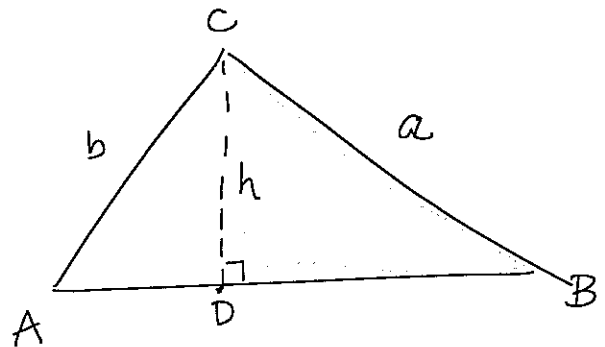
$\triangle BCD$

$$b \sin A = \frac{h}{b} \cdot b \quad a \sin B = \frac{h}{a} \cdot a$$

$$b \sin A = h \quad a \sin B = h$$

$$\frac{b \sin A}{ab} = \frac{a \sin B}{ab}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$



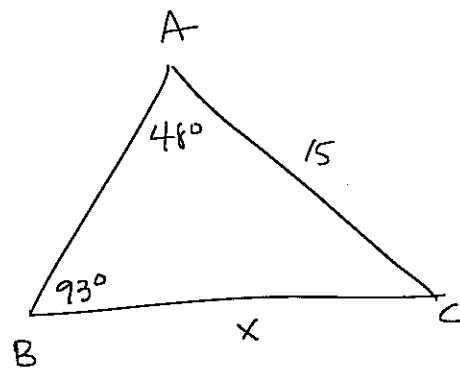
Example #1: Using the Law of Sines (AAS)

In $\triangle ABC$, $m\angle A = 48^\circ$, $m\angle B = 93^\circ$, and $AC = 15$. Find BC to the nearest tenth.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 48}{x} = \frac{\sin 93}{15}$$

$$x = \frac{(15 \sin 48)}{(\sin 93)} \approx 11.2$$



Example #2: Using the Law of Sines (SSA)

In $\triangle RST$, $RT = 11$, $ST = 18$, and $m\angle R = 120^\circ$. Find $m\angle S$ to the nearest tenth.

$$\frac{\sin R}{r} = \frac{\sin S}{s}$$

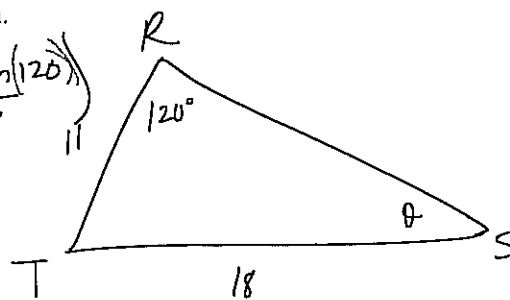
$$\frac{\sin 120}{18} = \frac{\sin \theta}{11}$$

$$11 \frac{\sin 120}{18} = \frac{18 \sin \theta}{18}$$

$$\frac{11 \sin 120}{18} = \sin \theta$$

$$\theta = \sin^{-1} \left(\frac{11 \sin(120)}{18} \right)$$

$$\theta \approx 32.0^\circ$$



Example #3:

A ship has been at sea longer than expected and has only enough fuel to safely sail another 42 miles. Port City Lighthouse and Cove Town Lighthouse are located 40 miles apart along the coast. At sea, the captain cannot determine the distances by observation. The triangle formed by the lighthouses and the ship is shown. Can the ship sail safely to either lighthouse?

$$\frac{\sin 58}{40} = \frac{\sin 70}{P} = \frac{\sin 52}{C}$$

$$\frac{\sin 58}{40} = \frac{\sin 70}{P}$$

$$\text{and } \frac{\sin 58}{40} = \frac{\sin 52}{C}$$

$$\frac{P \sin 58}{\sin 58} = \frac{40 \sin 70}{\sin 58}$$

$$\frac{C \sin 58}{\sin 58} = \frac{40 \sin 52}{\sin 58}$$

$$P = \frac{40 \sin 70}{\sin 58}$$

$$C = \frac{40 \sin 52}{\sin 58}$$

$$P \approx 44.3 \text{ mi}$$

$$C \approx 37.2 \text{ mi}$$

